UNIVERSITY OF PUNE

DEPARTMENT OF MATHEMATICS

SYLLABUS

Second Year M.Sc
Optional Subjects
MT 01 - OPERATIONS RESEARCH

Unit I - Kuhn – Tucker conditions of Optimality – Quadratic Programming
(Sections 19.2.2B, 20.2.2)

Unit II - Inventory Models
(Sections 14.1 to 14.3)

Unit III - Queuing Models
(Section 15.1, 15.2, 15.4, 15.5)

Unit IV - Project Scheduling By PERT – CPM
(Sections 13.1 to 13.4)

Unit V - Simulation Modeling with SIMNET – II
(Sections 17.1 to 17.10)

MT 02- INTEGRAL TRANSFORMS

A) Classification of Linear Integral Equations: Fredholm, Volterra, Integro-Differential Equations, Singular Integral Equations, Converting Volterra Equation to ODE, Conversion of IVP to Volterra equation Conversion of BVP to Fredholm equation


C) Volterra Integral Equation - Adomian Decomposition method, Series solution method, converting Volterra equation to VIP, Successive Approximation method, successive substitution method, comparison between alternative methods.


F) Non Linear Integral Equations - Non linear Fredholm Integral equations, Direct Computation, decomposition method, Non linear Volterra Integral Equation, Series solution, Decomposition method

G) Existence and uniqueness of solutions using fixed-point theorems in case of Linear and nonlinear Volterra and Fredholm integral equations.


Reference Books:


MT 03 - NUMBER THEORY


3. Quadratic Residues, Quadratic Reciprocity.


5. Diophantine equations. The equation \( ax + by = c \), Pythagorean triangles, Assorted examples. Rational points on curves.


MT 04 - CODING THEORY

1. Error detection: correction and decoding: Communication channels, Maximum likelihood decoding, Hamming distance, Nearest neighbor / minimum distance decoding, Distance of a code.

2. Linear codes: Vector spaces over finite fields, Linear codes, Hamming weight, Bases of linear codes, Generator matrix and parity check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes, Cossets, Nearest neighbor decoding for linear codes, Syndrome decoding.

3. Cyclic codes: Definitions, Generator polynomials, Generator and parity check matrices, Decoding of cyclic codes, Burst-error-correcting codes.

4. Some special cyclic codes: BCH codes, Definitions, Parameters of BCH codes, Decoding of BCH codes.

Reference: 1. San Ling and Chaoing xing, Coding Theory- A First Course
MT 05 - GRAPH THEORY

1. Fundamental concepts : Definitions and examples, graphs as models, matrices and isomorphism, paths, connected graphs, bipartite graphs, extremality vertex degree, the Pigeonhole principal, Turan’s theorem, degree sequences, graphic sequences, degree and digraphs.

2. Tree and Distances : Properties of tree, distance in graphs, stronger results, disjoint spanning trees, shortest paths, trees in computer science, Eulerian circuits.

3. Matching and Factors : Matching in bipartite graphs, maximum matchings, Hall’s matching conditions, Min-Matching in bipartite graphs, sets, applications and algorithms, maximum bipartite matching, weighted bipartite matching, in general graphs, Tutte’s 1-factor theorem, factors of graphs.


Prescribed Book :
Douglas B. West, Introduction to Graph Theory Prentice- Hall, New Delhi (1999)

Reference Books:
1. John Clarke and D.A. Holton, A First Look at Graph Theory, Allied Publisher (1991)
MT 06 - LATTICE THEORY

Two definitions of lattices, Hasse diagrams, homomorphism, isotone maps, ideals, congruence relations, congruence lattices, the homomorphism theorem, product of lattices, complete lattice, ideal lattice, distributive –modular inequalities and identifies, complements, pseudocomplements, Boolean lattice of pseudocomplements, join and meet-irreducible elements.

Characterization theorems and representation theorems-Dedekind’s modularity criterion Birkhoff’s distributivity criterion, hereditary subsets, rings of sets, Stone theorems, Nachbin theorem, statements of Hashimoto’s theorem.

Modular lattices, isomorphism theorem, Upper and lower covering conditions, Kurosu-Ore theorem, independent sets (Drops results involving projectivity and sublattice generated by sets / elements)

Semimodular lattices Jordan-Holder chain condition, Modular pair, M-symmetric lattices.

Prescribed Book:

General Lattice Theory

Author- G. Gratzer (Birkhauser, IInd Edition 1998)

Chap. 1 Section 1,2,3,4,6, Cha. 2 Section-1, Chap.3. Section –1,2.
1. Geometric primitives [Chap. 1]
2. Line intersection [Chaps. 2] plus randomized incremental
3. Triangulation and visibility and [Chaps. 3,15]
4. Linear programming in two and three dimensions [Chap. 4]
5. Orthogonal range searching [Chaps. 5,10]
6. Point location and Binary Space Partitions [Chaps. 6,12]
7. Voronoi diagrams and Delaunay triangulation [Chaps. 7,9]
8. Convex hulls [Chap. 11]
9. Non-orthogonal range searching [Chap. 16]
10. Curved Elements (Bezier, B-Splines)
11. Curve Reconstruction (reconstruction a curve(surface) from sample points)
12. 3-Dimensional Geometry

Prescribed Book:

MT 08 - CRYPTOGRAPHY

1. Introduction : Overview of course, Classical cryptography [parts of Chapter 1].


3. Public Key Encryption : Factoring and the RSA encryption [Chapter 4.1 - 4.4], Discrete log. Diffie-Hellman Key Exchange [Chapter 8.4 (only pages 270-273)]. ElGamal encryption [Chapter 5 (only pages 162-164)] , Digital Signatures [Chapter 6 (excluding 6.5 - 6.6)], One-time signatures, Rabin and ElGamal signatures schemes, Digital Signature Standard (DSS).


Prescribed Book :

MT 09 - FINANCIAL MATHEMATICS

1. Introduction to options and markets: types of options, interest rates and present values.

2. Black Sholes model: arbitrage, option values, pay offs and strategies, put call parity, Black Scholes equation, similarity solution and exact formulae for European options, American option, call and put options, free boundary problem.


4. Monte Carlo simulation: valuation by simulation

5. Finite difference methods: explicit and implicit methods with stability and conversions analysis methods for American options- constrained matrix problem, projected SOR, time stepping algorithms with convergence and numerical examples.

6. Lab component: implementation of the option pricing algorithms and evaluations for Indian companies.

Prescribed Book:


MT 10 - MODELLING AND SIMULATION

1. Introduction to modeling and simulation. System analysis, classification of systems. System theory basics, its relation to simulation.


3. Simulation systems and languages, means for model and experiment description. Principles of simulation system design


5. Continuous systems modeling. Overview of numerical methods used for continuous simulation.

6. Combined simulation. The role of simulation in digital systems design

7. Special model classes, models of heterogeneous systems.

8. Checking model validity, verification of models. Analysis of simulation results

9. Simulation results visualization. Interactive simulation

10. Design and control of simulation experiments. Model optimization

11. Generating, transformation, and testing of pseudorandom numbers. Stochastic models, Monte Carlo method

12. Overview of commonly used simulation systems.

Prescribed Book:


5. Common Sense Reasoning: No monotonic reasoning and modal logics for nonmonotonic reasoning. How to deal with Agents and their Beliefs.


Prescribed Book:
MT 12 - SYMMETRIES

1. Symmetry of plane figures of motions of the plan, finite groups of motions, discrete
groups of motion, symmetry, cosets, counting formula, permutation representations,
finite subgroups of the generators and relations

2. Operation of a group on itself, class equation of the isocahedral groups operations on
subsets groups of order 12, free group generators and relations.

3. Bilinear forms, symmetric forms, orthogonality, geometry associated to a positive
from, Hermitian forms, spectral theorem, conics and quadrics, normal operators, skew
symmetric forms.

Prescribed Book:

Artin : Algebra (Prentice-Hall)

Chapters 5, 6 (sections 1, 2, 3), 7.
MT 13 - WAVELET ANALYSIS

1. Fourier Transform: Fourier transform on $L^1(\mathbb{R})$ and $L^2(\mathbb{R})$ and basic properties and examples

2. Windowed Fourier Transform: Motivation and definition of Windowed Fourier Transform and examples, Time frequency localization, the reconstruction formula.

3. Continuous Wavelet Transform: Motivation and Definition of the wavelet transform and examples, Basic properties, The reconstruction formula, Frequency localization, Orthonormal Wavelets.


Prescribed Book:
MT 14 - COMBINATORICS


2. Generating function: Generating function models, calculating of generating functions, partitions exponential generating functions, a summation method.

3. Recurrence Relations: Recurrence relation model, divide and conquer relations, solution of inhomogeneous recurrence relation, solution with generating functions.

4. Inclusion-exclusion: Counting with Venn diagrams inclusion formula, restricted positions and rook polynomials.

5. Ramsey Theory: Ramsey theorem, applications to geometrical problems.

Prescribed Book:


MT 15 - PARTIAL DIFFERENTIAL EQUATIONS

1. First order PDE, Linear Equations of first order, Charpit’s method, Jacobi’s method, Quasi-linear equations, Non-linear first order PDE.

2. Second ordered PDE: Genesis, Classification, One dimensional Wave equation, Laplace equation, Boundary Value Problems, Maximum and Minimum Principles, Cauchy Problem,

3. Heat Conduction Problem, Duhamel’s Principle

Prescribed Book :

1. Fuzzy Sets and Operations on Them
2. Fuzzy Relations
3. Fuzzy Rules
4. Approximate Reasoning
5. Fuzzy Logic
6. Fuzzy Systems (e.g., Fuzzy Logic Control)
7. Fuzzy Logic in Pattern Recognition
8. Fuzzy Decision Making
9. Fuzzy Logic Applications

Prescribed Book:


MT 17 - STATISTICS AND PROBABILITY

1. Introduction to Discrete Probability: Intuitive concepts: probability of an event as a measure between 0 and 1; random variable; probability distribution; frequency interpretation of probability; random numbers; coins, dice, and other games; simulations; odds; historical development of probability; random walks.

2. Formal concepts: sample space, outcomes, and events; random variable; discrete distribution functions and axioms of probability; unions, intersections, and complements; properties of probabilities, principle of inclusion and exclusion; tree diagrams; uniform distributions over finite sets, symmetry; infinite sample spaces with discrete probabilities.

3. Introduction to Continuous Probability: The intuitive problems with probabilities over space (line, plane, \( \mathbb{R}^n \) in general). Monte Carlo simulations, Buffon's needle. Formal concepts: density function for a continuous random variable; integration; cumulative distribution functions; derivatives; exponential density function;

4. Conditional Probability: Intuitive concept of conditional probability; formal definition of conditional probability; Bayes' formula for inverting conditional probabilities; independent events; joint distribution functions; independent random variables; independent trials. Conditional density functions for continuous distributions; the beta distribution

5. Distributions and Densities: Uniform continuous distributions; geometric distribution; Poisson distribution; exponential and gamma distributions; introduction to queuing theory; normal (Gaussian) distribution; Chi-squared distribution

6. Expected Value and Variance: Expected value for discrete random variables, expectation; linearity of expectation; expectation of independent random variables; conditional expectation; variance and standard deviation; variance of various distributions. Expectation and variance for continuous random variables.

7. Sums of Random Variables: Analysis of sums of independent random variables with identical distributions, that is, independent trials.

8. Law of Large Numbers: Chebychev inequality, law of averages, law of large numbers.

9. The Central Limit Theorem: The central limit theorem for Bernoulli trials, binomial distributions again, the normal distribution, the general central limit theorem.

Prescribed Book:

MT 18 - FLUID DYNAMICS

1. Physical Properties of fluids. Concept of fluids, Continuum Hypothesis, density, specific weight, specific volume.

2. Kinematics of Fluids: Eulerian and Lagrangian methods of description of fluids, Equivalence of Eulerian and Lagrangian method, General motion of fluid element, integrability and compatibility conditions, strain rate tensor, stream line, path line, streak lines, stream function, vortex lines, circulation.

3. Stresses in Fluids: Stress tensor, symmetry of stress tensor, transformation of stress components from one co-ordinate system to another, principle axes and principle values of stress tensor.


5. Irrotational and Rotational Flows: Bernoulli’s equation, Bernoulli’s equation for irrotational flows, Two dimensional irrotational incompressible flows, Blasius theorem, Circle theorem, sources and sinks, sources sinks and doublets in two dimensional flows, methods of images.

Prescribed Book:
1. Relatively compact sets, compactly continuous operators, finite dimensional operators, transformation that is bounded but not completely continuous, a type of transformation that is always completely continuous, further properties of completely continuous transformations.

2. Spectra and the resolvent set, Approximate proper values, Banach Algebra With identity, compactness of the spectrum, the resolvent operator, Spectral radius and spectral mapping theorem for polynomials, the Gelfand Theory.

3. Sesquilinear functions: Spectral results for normal and completely continuous operators, numerical range

4. The Fredholm alternative theory, the spectral theorem for bounded, normal Finite dimensional operators.

5. Commutative Banach Algebras, ideals and homomorphisms.

Prescribed Book:
MT 20 - BOUNDARY VALUE PROBLEMS

1. Definition of boundary Value Problems, the heat equation, wave equation, Laplace’s equation, the Fourier method, Liner Operators, Principal of Superposition, series solutions, uniform convergence (weierstrass M-test), separation of variables, non homogeneous conditions, Sturm-Liouville problems, formal solutions, the vibrating string.

2. Orthogonal sets of functions, Generalized Fourier series, Best approximation in the mean, Convergence in the mean, the orthonormal trigonometric functions, other types of orthogonality.


4. Bessel function $J_n$, recurrence relation, the zero of $J_0 (X)$ and related functions, Fourier-Bessel series, Temperatures in a long cylinder.

5. Legendre polynomials, orthogonality of Legendre polynomials, Legendre series, Dirichlet Problem in spherical regions.

Prescribed Text Book

1. Rings with involution
2. Poset of projections
3. Proper involutions and C*-algebras
4. Rickart *-rings and Bear *-ring
5. Weakly Rickart *-rings and unitification
6. Central cover
7. Additivity of projections
8. Comparability axioms and parallelogram law
9. Finite and abelian projections
10. Structure Theorem

Prescribed Book: Bear *-rings, S.K. Berberian, Springer
MT 22 - MATROID THEORY

1. Basic definitions and examples

Independent sets and circuits, bases, rank, closure, geometric representations of matroids of small rank, transversal matroids, the lattice of flats, the greedy algorithm.

2. Duality

The definition and basic properties, duals of representable matroids, duals of graphic matroids, duals of traversal matroids.

3. Minors

Contraction, Minors of certain matroids, flats and the sum theorem

4. Connectivity

Connectivity, for graphs and matroids, properties of matroid connectivity, more properties of connectivity.

Prescribed Book:
James G. Oxley, Matroid Theory Science Publications, Oxford (1992)(Chapter 1 to 4)
MT 23 - SPERNER THEORY

1. Introduction and Sperner`s Theorem: A Simple intersection result, Sperner`s theorem, Theorem of Bollobaas.

2. Normalized Matching and rank numbers: Sperner`s proof, system of distinct representatives, L Y M inequalities, and normalized matching property. Rank numbers, some examples.

3. Symmetric Chain: Symmetric chain decompositions, Dilworth`s theorem, symmetric chains of sets, Application to Nested chains, posets with symmetric chain decompositions.

4. Rank numbers of multisets. Unimodality and log connectivity, the normalized matching property. The largest size of a rank number.

Prescribed Text Book

Ian Anderson : Combinatorics of Finite Sets. (Oxford Science Publications)

Reference Book.

Konrad Engel: Sperner Theory (Cambridge University Press)

2) Nonlinear Systems: Local Theory, Fundamental existence theorem dependence on initial conditions and parameters, the maximal interval of existence, Flow defined by a differential equation. Linearization, stable manifold theorem, Hartman-Grobman theorem, Stability and Lipunov functions, Saddles, Nodes, Foci and centers, Nonhyperbolic critical points in $\mathbb{R}^n$, Gradient and Hamiltonian system.

Text (1) L. Perko- Differential Equations and Dynamical systems (1991) Springer-verlag

1. Sec 1.1-1.6 Survey of Elementary Principles.
2. Sec. 2.1-2.7 Variational Principles & Lagrange`s Equation
3. Sec.3.1-3.7 Central Force problem
4. Sec. 4.1-4.10 Kinematics of rigid body motion
5. Sec. 8.1-8.2 Hamilton Equations of motion
6. Sec.9.1-9.9 Canonical Transformations

Prescribed Book :